LINEAR ALGEBRAIC APPROACH TO H-BASIS COMPUTATION

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Macaulay introduced the notion of H-bases [2]. His original motivation was the transformation of systems of polynomial equations into simpler ones. The power of this concept was not really understood presumably because of the lack of facilities for symbolic computations. Later Buchberger invented Gröbner bases for computing multiplication tables for factor rings [1]. When Computer Algebra Systems came up, Buchberger's Algorithm for computing Gröbner basis is implemented to this programs. H-bases have not been used in computational problems as extensively as Gröbner basis. However, Möller and Sauer show that H-bases yield a perfect replacement for the Gröbner bases in some Numerical Analysis problems, see [([3, 4, 5])]. All approaches related to Gröbner Bases are fundamentally tied to on term orders which leads to asymmetry among the variables to be considered. On the other hand, the concept of H-bases is based solely on homogeneous terms of a polynomial. Hence H-bases lead to a significant stabilization of the computations when they used instead of Gröbner bases.

It is well known that a Gröbner basis with respect to a degree compatible ordering is an H-basis as well. However, this Gröbner basis may contain some unnecessary elements. Yılmaz and Kılıçarslan [6] gave a method of eliminating of these unnecessary elements during the Gröbner basis computation. H-basis have not been preferred over Gröbner basis because there is no general algorithm for computing it. An algorithm involving only linear algebraic computations is proposed by Moller and Thomas but it relies on the a priori knowledge of a system of generator of the syzygy module which cannot be expected to be known in advance in most situations (see [3]). The only known method for computation of a basis of syzygy module depends on Gröbner basis computation.

In this study, we give a method for computation of module of syzygies using only linear algebraic techniques. Our method computes module of syzygies only for homogeneous ideals. This is enough to give an algorithm for obtaining H-basis with only some linear algebraic and combinatorial computations when it is combined with Möller and Thomas' idea.

References

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