

Bézout Matrices and Complex Roots of Quaternion Polynomials

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The notion of Bézout matrix is introduced by Sylvester (1853) and Cayley (1857). It is a special square matrix associated with two polynomials with very useful properties. Thus, it is an essential tool in studying broad variety of topics: zeros of polynomials, stability of differential equations, rational transformations of algebraic curves, etc. In this paper, we use Bézout matrices in order to study the complex roots of polynomials with quaternion coefficients. More precisely, quaternion polynomials have two kind of roots: isolated and spherical. A spherical root generates a class of roots which contains only one complex number z and its conjugate \bar{z} , and this class can be determined by z . Using Bézout matrices, we give necessary and sufficient conditions, for a quaternion polynomial to have a complex root, a spherical root, and a complex isolated root. These results are applied in the study of Rational Rotation Minimizing Frame Curves which are useful in many applications, as robotics, computer graphics, motion design and control in computer animation, swept surfaces construction etc.

Keywords: Quaternion polynomial; Bézout Matrices; Spherical Root; Isolated Root.

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